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# V-Tree: Efficient kNN Search on Moving Objects with Road-Network Constraints

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***Abstract*—Intelligent transportation systems, e.g., Uber, have become an important tool for urban transportation. An important problem is $k$ nearest neighbor (kNN) search on moving objects with road-network constraints, which, given moving objects on the road networks and a query, finds $k$ nearest objects to the query location. Existing studies focus on either kNN search on static objects or continuous kNN search with Euclidean-distance constraints. The former cannot support dynamic updates of moving objects while the latter cannot support road networks. Since the objects are dynamically moving on the road networks, there are two main challenges. The first is how to index the moving objects on road networks and the second is how to find the $k$ nearest moving objects. To address these challenges, in this paper we proposes a new index, `V-Tree`, which has two salient features. Firstly, it is a balanced search tree and can support efficient kNN search. Secondly, it can support dynamical updates of moving objects. To build a `V-Tree`, we iteratively partition the road network into sub-networks and build a tree structure on top of the sub-networks. Then we associate the moving objects on their nearest vertices in the `V-Tree`. When the location of an object is updated, we only need to update the tree nodes on the path from the corresponding leaf node to the root. We design a novel kNN search algorithm using `V-Tree` by pruning large numbers of irrelevant vertices in the road network. Experimental results on real datasets show that our method significantly outperforms baseline approaches by 2-3 orders of magnitude.**

## I. Introduction

Intelligent transportation systems, e.g., Uber, have been emerged as an important transportation tool. For drivers, Uber represents a flexible new way to earn money. For cities, Uber helps strengthen local economies, improves access to transportation, and makes streets safer. For passengers, Uber helps them easy to get a cab. Thus Uber has been widely used in our daily life.

An important problem in Uber is $K$ nearest neighbor (kNN) search on moving objects on road networks, which finds the $k$ nearest objects to a given query location. Existing studies focus on either kNN search on static objects [22], [16], [25], [10], [31], [30], [22] or continuous kNN search with Euclidean distance constraint [26], [11], [23], [27], [5], [9], [6], [17], [28], [29], [8]. The former cannot support dynamic updates of moving objects, because it is rather expensive to update the index. The latter cannot efficiently compute the distance on road networks. Thus they cannot efficiently address this problem. For example, Uber in China took more than 180 seconds to find the kNN results for each query.

Two factors make the problem more challenging. Firstly, the objects are dynamically moving on the road networks. For example, there are more than 60K taxies in Beijing and the locations of cars are updated every second. Thus one challenge is how to index the moving objects on road networks. Secondly, there are lots of queries. For example, in Beijing there are 1 million queries each day and in the peak time there are 100K queries in each second. Thus another challenge is how to find the kNN moving objects efficiently.

To address these challenges, in this paper we proposes a new index, `V-Tree`, which has two salient features. Firstly, it is a balanced search tree and can support efficient kNN search. Secondly, it can support dynamical updates of moving objects. To achieve this goal, we iteratively partition the road network into sub-networks and build a tree structure on top of the sub-networks, where the tree nodes are sub-networks. Then we associate the moving objects to their closet vertices on the road networks. To facilitate the kNN computation, we also keep the shortest distances from some important vertices (called borders) to the vertices with associated objects. When the location of an object is updated, we only need to update the tree nodes on the path from the corresponding leaf node to the root. We also design a novel kNN search algorithm using the borders to efficiently compute $k$ nearest objects, which adopts a best-first method and can prune many irrelevant objects.

To summarize, we make the following contributions.

1) We devise an efficient and scalable tree index for moving objects on road network, called `V-Tree`. The space complexity of `V-Tree` is $\mathcal{O}(\log|V|\cdot|V|)$, where $|V|$ is the number of vertices in the road network.
2) We propose an efficient update strategy to support updates of moving objects. The average time complexity is $O(\frac{|V|\min(\log|\mathcal{M}|,\log|V|)}{|\mathcal{M}|})$, where $|\mathcal{M}|$ is the number of objects moving on the road network.
3) We devise a novel kNN search method using `V-Tree` to compute $k$ nearest objects. The average time complexity is $O(\frac{k\cdot|V|\min(\log|\mathcal{M}|,\log|V|)}{|\mathcal{M}|})$.
4) We have conducted extensive experiments to evaluate our method on real datasets. Experimental results show that our method significantly outperforms baseline approaches by 2 orders of magnitude. We also publicize our source code at https://github.com/TsinghuaDatabaseGroup/VTree.

The structure of this paper is organized as follows. We first formulate the problem in Section II. The `V-Tree` is proposed in Section III, and we devise an efficient kNN search algorithm in Section IV. Experimental results are reported in Section V. We review related work in Section VI and conclude the paper in Section VII.

## II. Preliminaries

**Road Network.** We model a road network as a directed weighted graph $G=\langle V,E\rangle$, where $V$ is a set of vertices and $E$ is a set of edges. Each edge $(u,v)\in E$ $(u,v\in V)$

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has a weight, which is the travel cost from $u$ to $v$ (e.g., distance, travel time.). Given a path from $u$ to $v$, the *distance* of this path is the sum of weights of the edges along the path. Let $\texttt{SPath}(u, v)$ denote the shortest path from $u$ to $v$ and $\texttt{SPDist}(u, v)$ denote the shortest-path distance. We use graph and road network interchangeably for ease of presentation.

**Moving Objects.** Each object (e.g., vehicle) moving on the road network is represented by $m = \langle t, p\rangle$, where $p$ is the geo-location of $m$ at time $t$. Note the locations of objects are updated periodically (e.g., every second).

We suppose moving object is driving on the road. We can utilize existing techniques [4], [24], [19] to map an object to an edge on the road. Suppose $m$ is driving on edge $e = (u, v)$ and its distance to $v$ is $\delta = \texttt{SPDist}(m, v)$. We use $m = (t, (u, v), \delta)$ to denote the object. The shortest-path distance from a moving object $m$ to a vertex $x$, denoted by $\texttt{SPDist}(m, x)$, can be computed by $\delta + \texttt{SPDist}(v, x)$. In Figure 1, the distance from $m_1$ to $v_7$ is computed by summing up the $\delta$ from $m_1$ to $v_5$, and $\texttt{SPDist}(v_5, v_7)$, i.e., $\texttt{SPDist}(m_1, v_7) = 80 + 180 = 260$.

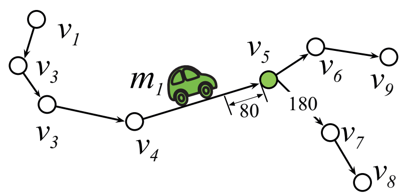



Fig. 1. Example of Objects.

**kNN Query.** Given a graph $DAG$, a moving object set $\mathcal{M}$, a kNN query $q = \langle v, k\rangle$, where $v$ is a query location, and $k$ is an integer. The answer of $q$ is a set of $k$ nearest objects to the query location such that.
(1) The size of $\mathcal{R}$ is $k$, i.e., $|\mathcal{R}| = k$;
(2) Each answer is an object, i.e., $\mathcal{R} \subseteq \mathcal{M}$.
(3) $\forall x \in \mathcal{R}, \forall y \in \mathcal{M} - \mathcal{R}, \texttt{SPDist}(v, x) \leq \texttt{SPDist}(v, y)$.

Similar to existing works [4], [24], [19], we assume that the query location of $q$ is at a vertex. If the query location is not at a vertex, then $(i)$ if it is on an edge, we find the top-$k$ answers to the two vertices of edge $e$, and then select the top-$k$ answers from these $2k$ candidates; $(ii)$ if it is not on an edge, we find the closest edge $e$ to the query location using existing technique [4], [24], [19] and then utilize the method in case $(i)$ to compute the top-$k$ answers. Thus in the paper, we focus on the case that the query location is at a vertex.

## III. The **V-Tree** Index

We propose a tree index to support kNN search on moving objects, called `V-Tree`. We first formally define `V-Tree` (Section III-A) and then discuss how to construct `V-Tree` (Section III-B). Next we present utilizing `V-Tree` to compute shortest-path distance (Section III-C). We close this section by discussing how to update `V-Tree` (Section III-D).

### *A.* **V-Tree**

Before we introduce the `V-Tree` structure, we first define some concepts.

*Definition 1:* (Graph Partition). Given a graph $G = \langle V, E\rangle$, where $V$ is the vertex set and $E$ is the edge set of $G$, $f$ is the fanout, we partition $G$ into $f$ subgraphs, i.e., $G_1 = \langle V_1, E_1\rangle, G_2 = \langle V_2, E_2\rangle, ..., G_f = \langle V_f, E_f\rangle$, such that
(1) Completeness on vertices: $\cup_{1\leq i\leq f} V_i = V$;
(2) Disjoint on vertices: For $i \neq j, V_i \cap V_j = \phi$; and
(3) Completeness on edges for vertices in the same subgraph: $\forall u, v \in V_i$, if $(u, v) \in E$, $(u, v) \in E_i$.

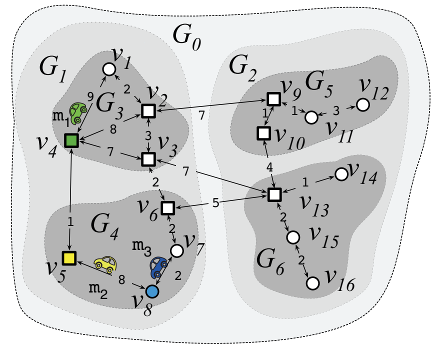



Fig. 2. Road Network Partition.

We build a tree hierarchy based on the graph partition. Initially the root is the graph $G$. Then we iteratively partition the graph $G$ as follows. We partition $G$ into $f$ subgraphs $G_1$, $G_2$,..., $G_f$, which are taken as $G$'s children. These $f$ subgraphs are called *child subgraphs* of $G$. The *child-subgraph* set of $G$ is denoted by $\mathcal{C}(G)$. $G$ is called the *parent graph* of its *child subgraphs*. The *parent graph* of $G_i$ is denoted by $G_i^p$. Next we iteratively partition $G_i$, and take its child subgraphs as its children. We terminate if $G_i$ has less than $\tau$ vertices, where $\tau$ is a threshold. Figure 2 shows an example of this iterative graph partition process, with an original road network graph $G_0 = G$, $f = 2$ and $\tau = 4$. $G_0$ is partitioned into two child subgraphs $G_1$ and $G_2$, so $\mathcal{C}(G_0) = (G_1, G_2)$ and $G_1^p = G_0$.

*Definition 2 (Boundary Vertex):* Given a graph $G = \langle V, E\rangle$, its subgraph $G_i$, a vertex $\beta_j$ in $G_i$ is called a boundary vertex if $\exists(\beta_j, v) \in E$ and $v$ is not in $G_i$.The boundary vertex set of $G_i$ is denoted by $B(G_i)$.

Given two vertices $u$ and $v$ in $G$, if $v \in G_i$ and $u \notin G_i$, then the shortest path from $u$ to $v$ must bypass boundary vertices of $G_i$, because if $v$ connects to $u$, it must go out $G_i$ (i.e., bypass a boundary vertex) as stated in Lemma 1 .

*Lemma 1:* Given a subgraph $G_i = \langle V_i, E_i\rangle$ and two vertices $v_i \in V_i$, $v_j \notin V_i$, the shortest path from $v_j$ to $v_i$ must contain a boundary vertex $\beta$ in $G_i$ such that $\texttt{SPDist}(v_j, v_i) = \texttt{SPDist}(v_j, \beta) + \texttt{SPDist}(\beta, v_i)$.

*Proof:* The proofs of Lemmas and Theorems can be found in our technical report [1]. ∎

Accordingly, given two subgraphs $G_i, G_j$ and two vertices $v_i \in G_i$ and $v_j \in G_j$, there must exist two boundary vertices $\beta_i \in G_i$ and $\beta_j \in G_j$ such that

$$\texttt{SPDist}(v_j, v_i) = \texttt{SPDist}(v_j, \beta_j) + \texttt{SPDist}(\beta_j, \beta_i) + \texttt{SPDist}(\beta_i, v_i).$$

To efficiently compute the shortest-path distance, we can precompute the distances between vertices and boundary vertices, and between boundary vertices and boundary vertices. However this involves huge storage space. To address this issue, we only precompute the shortest-path distances between vertices for leaf nodes, and the shortest-path distances between boundary vertices and boundary vertices for non-leaf nodes with the same parents. We will show that the shortest path between two vertices can be efficiently computed based on these precomputed distances in Section III-C.

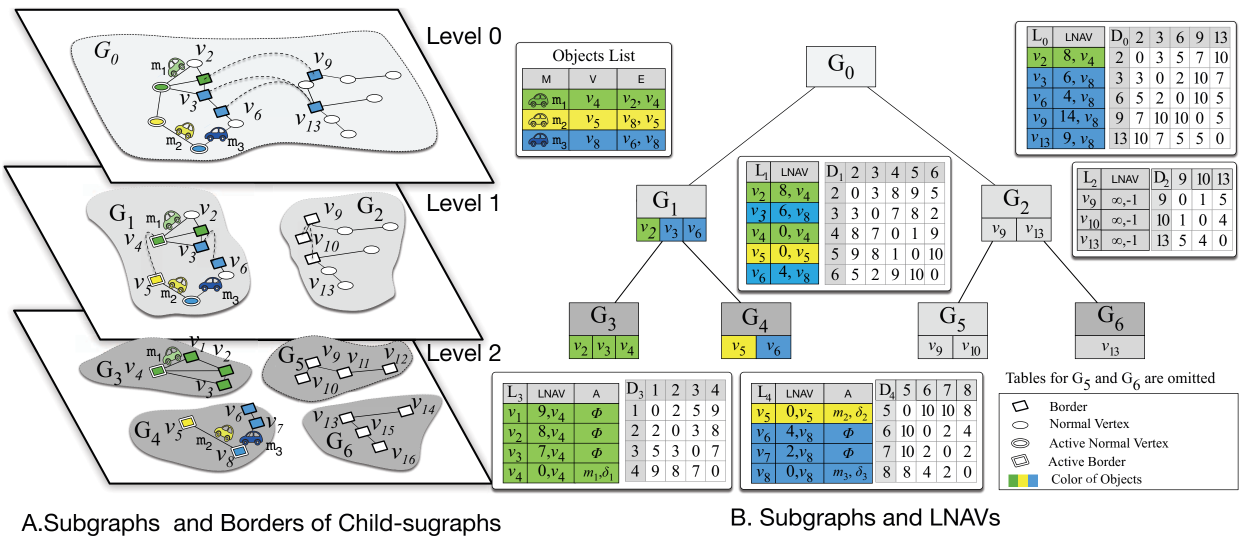



Fig. 3. V-Tree Structure (Square Vertices are Borders of Child-subgraphs in Different Levels).

**Distance Matrix.** Each leaf subgraph maintains a distance matrix $\mathcal{D}$ of all vertices in the subgraph, i.e., $\mathcal{D}_{i,j} = \texttt{SPDist}(v_i, v_j)$ for all $v_i$ and $v_j$ in the subgraph. Each non-leaf subgraph maintains a distance matrix $\mathcal{D}$ of all boundary vertices of its child subgraphs. i.e., $\mathcal{D}_{i,j} = \texttt{SPDist}(\beta_i, \beta_j)$, where $\beta_i, \beta_j$ are boundary vertices of its child subgraphs. For ease of presentation, let $\mathbb{B}(G_i)$ denote the set of boundary vertices of $G_i$'s child subgraphs, and we call each boundary vertex in $\mathbb{B}(G_i)$ is a *border*.

*Definition 3 (Border):* Given a non-leaf subgraph $G_i$, a vertex $\beta_i$ in $G_i$ is called a border if it is a boundary vertex of one of its child subgraph. Given a leaf subgraph, each of its boundary vertex is a border.

As shown in Figure 2, $B(G_3) = \{v_2, v_3, v_4\}$, $B(G_4) = \{v_5, v_6\}$, $\mathbb{B}(G_1) = \{v_2, v_3, v_4, v_5, v_6\}$. The shortest path from $v_1$ to $v_8$ bypasses borders $v_3$ of $G_1$ and $v_6$ of $G_4$.

Next, we discuss how to associate the moving objects into the vertices.

*Definition 4 (Active and Inactive Vertex):* Given an object $m$ on an edge $e$ which is moving to the vertex $v$, we call $m$ an $active$ object of vertex $v$ and call $v$ an active vertex. A vertex is $active$ if it has active objects; $inactive$ otherwise.

For example, in Figure 2, $m_1$ is on edge $e(v_1, v_4)$ and moving to $v_4$ with an offset of $0$. $m_1$ is an active object of $v_4$ and $v_4$ is an active vertex. Vertex $v_1$ is inactive as it has no active object. The distance of the shortest path from an object $m$ to any vertex $u \in G$, $\texttt{SPDist}(m, u)$, is the summation of the offset $\delta$ to vertex $v$ that $m$ is driving to and $\texttt{SPDist}(v, u)$, i.e., $\texttt{SPDist}(m, u) = \delta + \texttt{SPDist}(v, u)$.

*Definition 5 (Local Nearest Active Vertex-*`LNAV`*):* Given a vertex $u$ in a subgraph $G_i$, an active vertex $v$ in $G_i$ is called the local nearest active vertex (`LNAV`) if $v$ has the minimum distance to $u$ among all the active vertices in $G_i$.

For example, in Figure 2, $v_4$, $v_5$, and $v_8$ are active vertices. The `LNAV` of $v_3$ in $G_3$ is $v_4$; in $G_1$, the `LNAV` of $v_3$ is $v_8$.

Note that the global nearest active vertex (`GNAV`) of $u$ may be not in $G_i$. We use `LNAV` instead of `GNAV` because it is rather expensive to compute and update `GNAV` while `LNAV` is easy to maintain and update. More importantly, we can efficiently compute `GNAV` based on `LNAV` which will be discussed in Section IV.

`LNAV` **Table** $\mathcal{L}$**.** For each non-leaf subgraph $G_i$, we store the `LNAV` for all the borders in the subgraph in a `NAV` table, denoted by $\mathcal{L}_i$. The `LNAV` table has two columns: the `LNAV` and the distance to `LNAV`. For each border $\beta$ in subgraph $G_i$, $\mathcal{L}_i[\beta].\gamma$ and $\mathcal{L}_i[\beta].\delta$ keep the `NAV` of $\beta$ in $G_i$ and the distance to $\beta$, respectively.

For example, in Figure 2, $v_3$ is a boundary vertex of $G_3$, the `LNAV` to $v_3$ in $G_1$ is $v_8$, so $\mathcal{L}_1[v_3].\delta = 6$ and $\mathcal{L}_1[v_3].\gamma = v_8$. We will discuss later that the `LNAV` table in each subgraph can be updated efficiently in Section III-D.

**Active Object Table** $\mathcal{A}$**.** For each leaf subgraph, we maintain an active object table $\mathcal{A}$. For each vertex $v$ in the leaf subgraph $G_i$, we use $\mathcal{A}_i[v]$ to keep its active objects, where each entry is $\langle m, \delta \rangle$ to represent an active object and its offset to its active vertex.

Based on the above notations, next we are ready to define the `V-Tree`.

*Definition 6 (*`V-Tree`*):* A `V-Tree` of a road network $G$ is a balanced search tree that has the graph partition hierarchy and satisfies the following properties.

(1) Each node in `V-Tree` corresponds to a subgraph. Each non-leaf node has $f$ children. Each leaf node has less than $\tau$ vertices.

(2) Each node also maintains a distance matrix $\mathcal{D}$. Each leaf node maintains the distance matrix for its vertices while each non-leaf node maintains the distance matrix for its borders.

(3) Each node maintains a `LNAV` table $\mathcal{L}$. The leaf nodes maintain the `LNAV` for all vertices in the leaf nodes while the non-leaf nodes maintain the `LNAV` for its borders.

(4) Each leaf node maintains an active object table $\mathcal{A}$, which maintains the active objects for each vertex in the leaf node.

For example, Figure 3 shows the `V-Tree` of the road network and moving objects in Figure 2 with $f = 2$ and $\tau = 4$. Each non-leaf node stores a `LNAV` table (on the right or below the node). The rows in green are borders taking $v_4$ as the `LNAV`,

TABLE I. A LIST OF NOTATIONS USED IN THE PAPER

| Notation | Definition |
|---|---|
| $G = \langle V, E \rangle$ | Graph $G$ with vertex set $V$ and edge set $E$ |
| $\mathcal{M}$ | Set of Moving Objects |
| $B(G_i)$ | Boundary vertex set of $G_i$ |
| $\mathbb{B}(G_i)$ | *Border* set of $G_i$ |
| $\texttt{SPDist}(a, b)$ | Shortest path distance from $a$ to $b$ |
| $\mathcal{D}$ | Distance matrix |
| $\mathcal{D}_{i,j}$ | $\texttt{SPDist}(\beta_i, \beta_j)$ |
| $\mathcal{A}$ | Active object table |
| LNAV | Local nearest active vertex |
| $\mathcal{L}_i$ | LNAV table of $G_i$ |
| $\mathcal{L}_i[\beta].\gamma$ | NAV of $\beta$ in $G_i$ |
| $\mathcal{L}_i[\beta].\delta$ | The distance from $\mathcal{L}_i[\beta].\gamma$ to $\beta$ |
| $G_v^p$ | Parent graph of $G_v$ |
| $f$ | Fanout of V-Tree |
| $\tau$ | Maximum number of vertices in a leaf node |

to which $m_1$ is driving to. Similarly, the rows in yellow have $v_5$ as LNAV.

**Space Complexity of** V-Tree**.** Given a graph $G$ with $|V|$ vertices and $|\mathcal{M}|$ objects, the space complexity of V-Tree is $\mathcal{O}(|\mathcal{M}| + \log|V| \cdot |V|)$.

Table 1 summarizes a list of essential notations used in this paper. (Some notations will be introduced later.)

### *B.* V-Tree *Construction*

**Tree Hierarchy.** It aims not only to partition the graph to equal-size subgraphs, but more important to ensure the subgraphs have small size of borders in each level. Based on the planar separator theorem [18], given a planar graph with $|V|$ vertices, if we partition it into $f$ subgraphs, the number of boundary vertices of the subgraphs is $\mathcal{O}(\sqrt{|V|})$. We divide the full graph into $f$ equal-sized subgraphs by a famous multilevel algorithm [15], which can make each subgraph have almost the same size and a small number of borders. Specifically, we first partition the full-graph to $f$ equal-sized subgraphs. And for each subgraph, we divide it to $f$ equal-sized subgraphs. The partition terminates until the number of vertices contained in the subgraph is less than or equal to $\tau$.

**Distance Matrix $\mathcal{D}$.** It computes the shortest-path distances between all the borders for each non-leaf subgraphs and the shortest-path distances between all the vertices of the leaf subgraphs. The naive method is to compute the distance of vertices pair by pair. The construction complexity of this method is too high to scale to large road networks. To address this issue, we use the bottom-up method to compute the matrix by sharing the computations [31], [30], and the time complexity is $\mathcal{O}(|V|^{1.5})$.

**Active Object Table $\mathcal{A}$.** For a new object $m$, it adds $m$ into corresponding subgraph's active object table. The complexity is $\mathcal{O}(1)$.

LNAV **Table $\mathcal{L}$.** For a new object, we discuss how to add it into the LNAV table later.

For a V-Tree which does not contain any object, each LNAV entry in V-Tree is set to $\langle \infty, \phi \rangle$, and the active object of each vertex is $\phi$. The $\mathcal{A}$ and $\mathcal{L}$ tables will be updated when objects are added.

### *C. Implementing* SPDist *Function on* V-Tree

Given two vertices $u$ and $v$ on the road network, SPDist $(u, v)$ computes the shortest-path distance from $u$ to $v$. We consider the following two cases.

**Algorithm 1:** $\texttt{Add}(m, v)$

**Input**: $m, v$ // adding $m$ to vertex $v$

**if** $v.status$ = *inactive* **then**
  $\mathcal{L}_{G_v}[v] = \langle 0, v \rangle$;
  **for** *each vertex $u$ in $G_v$* **do**
    **if** $\mathcal{L}_{G_v}[u].\delta > \texttt{SPDist}(v, u)$ **then**
      $\mathcal{L}_{G_v}[u] \leftarrow \langle \texttt{SPDist}(v, u), v \rangle$;
  propagation $\leftarrow$ true;
  **while** $G_v \neq G_0$ *and* propagation **do**
    propagation $\leftarrow$ false ;
    **for** *each border $\beta$ in $\mathbb{B}(G_v^p) \cap \mathbb{B}(G_v)$* **do**
      **if** $\mathcal{L}_{G_v}[\beta].\delta < \mathcal{L}_{G_v^p}[\beta].\delta$ **then**
        $\mathcal{L}_{G_v^p}[\beta] \leftarrow \mathcal{L}_{G_v}[\beta]$;
        propagation $\leftarrow$ true;
    **for** *each border $\beta'$ in $\mathbb{B}(G_v^p) - \mathbb{B}(G_v)$* **do**
      $d \leftarrow \min_{\substack{\beta \in \mathbb{B}(G_v) \\ \mathcal{L}_{G_v^p}[\beta].\gamma = v}} \mathcal{L}_{G_v^p}[\beta].\delta + \texttt{SPDist}(\beta, \beta')$;
      **if** $\mathcal{L}_{G_v^p}[\beta'].\delta > d$ **then**
        $\mathcal{L}_{G_v^p}[\beta'] \leftarrow \langle d, v \rangle$;
    $G_v \leftarrow$ parent of $G_v$;
  $v.status \leftarrow$ active;
add $m$ to $\mathcal{A}[v]$ ;

$u, v$ **in the same leaf subgraph**. SPDist $(u, v)$ is directly gotten from the leaf distance matrix $\mathcal{D}[u][v]$. The time complexity of this case is $\mathcal{O}(1)$.

$u, v$ **in different leaf subgraphs**. We utilize a dynamic-programming algorithm [31] to compute SPDist $(u, v)$. The basic idea is as follows. We first location the leaf subgraphs $G_v$ and $G_u$ of $v$ and $u$ respectively, which can be implemented by a hash table. Then we compute the least common ancestor LCA of $G_v$ and $G_u$, and the nodes on the paths from LCA to $G_v$ and $G_u$. Next we enumerate the combinations of borders in these nodes to compute the shortest-path distance. We can utilize the dynamic-programming algorithm [31] to share the computations.

### *D. Updates on* V-Tree

As the road network will not change frequently, we focus on how to update the locations of active objects on V-Tree.

There are four cases for updating an active object $m$.

**Case 1.** Adding a new object $m$ on edge $(u, v)$ and driving to $v$. For example, an object, e.g., a taxi, becomes free from busy. We use algorithm $\texttt{Add}(m, v)$ to add $m$ to $v$.

**Case 2.** Deleting an object $m$ from edge $(u, v)$. For example, an object, e.g., a taxi, becomes busy from free. We use algorithm $\texttt{Del}(m, v)$ to delete $m$ from $v$.

**Case 3.** Object $m$ is still on edge $(u, v)$ and driving to $v$, but the offset from $m$ to $v$ is changed. Vertex $v$ is still an active vertex of $m$. We only need to update the offset of $m$ in $\mathcal{A}[v]$. The update complexity of this case is $\mathcal{O}$ (1).

**Case 4.** Object $m$ is moving to edge $(v, w)$ from edge $(u, v)$. $m$ is not an active object of $v$ and becomes an active object of $w$. We require to delete $m$ from $v$ and add $m$ to $w$. Thus we call the functions $\texttt{Del}(m, v)$ and $\texttt{Add}(m, w)$.

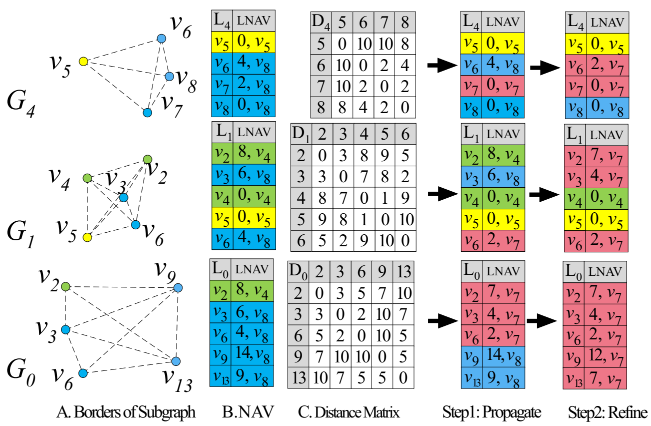


Fig. 4. Adding $m_4$ to $v_7$

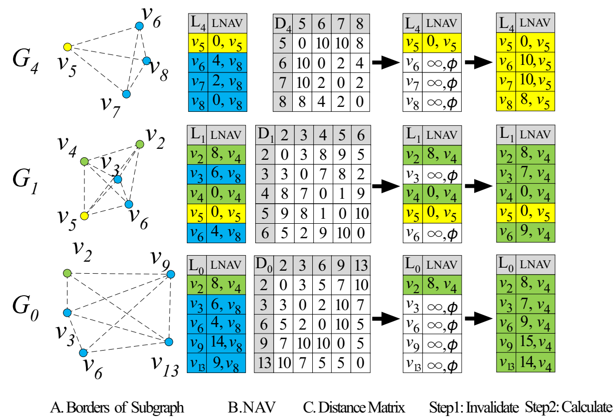


Fig. 5. Deleting $m_3$ from $v_8$

Thus we only need to consider two functions $\texttt{Del}(m, v)$ and $\texttt{Add}(m, v)$. Next we discuss how to implement these two functions.

*1) Adding $m$ to Vertex $v$, $\texttt{Add}(m, v)$:* There are two cases for $v$. $(i)$ Before adding $m$ to $v$, $v$ is already an active vertex, which already contains at least one object. The status of $v$ will not change. Thus all LNAV entries on V-Tree will not change. We just add $m$ into $\mathcal{A}[v]$. $(ii)$ Before adding $m$ to $v$, $v$ is an inactive vertex. The state of $v$ will change from inactive to active. We need to add $m$ into $\mathcal{A}[v]$. As $v$ may become the LNAV for some borders in the subgraphs containing $v$, we need to update the LNAV entries for such borders. Note $v$ will not become the LNAV for borders in the subgraphs that do not contain $v$. In this way, we only need to check the leaf node containing $v$, $G_v$, and its ancestors. To this end, we propose a bottom-up method to update the LNAV and the pseudo code is shown in Algorithm 1.

**Updating Leaf Graph $G_v$.** Consider a vertex $u$ in $G_v$ whose LNAV is not $v$, i.e., $\mathcal{L}_{G_v}[u].\gamma = w$ and $w \neq v$. If $\texttt{SPDist}(v, u) < \texttt{SPDist}(w, u)$, $v$ should be the LNAV of $u$ and we should update $\mathcal{L}[u]_{G_v}.\gamma$ to $v$ (lines 5-6 in Algorithm 1). Note that both $\texttt{SPDist}(v, u)$ and $\texttt{SPDist}(w, u)$ can be found from the pre-calculated distance matrix $\mathcal{D}$ in $G_v$.

**Updating The Ancestors of $G_v$.** If the LNAV of a border $\beta$ in $G_v$ is changed and $v$ becomes the LNAV in $G_v$, i.e., $\mathcal{L}_{G_v}[\beta].\gamma = v$, $v$ also becomes the LNAV of $\beta$ in $G_v$'s parent $G_v^p$, and thus we need to update the LNAV for its parent. The update may also influence other borders that are not in $G_v$, because it may shorten the distance from $v$ to such border. Next we propose a three-step method to update $G_v^p$.

*Step 1.* For each border $\beta$ in $\mathbb{B}(G_v^p) \cap \mathbb{B}(G_v)$, if $\mathcal{L}_{G_v}[\beta].\delta < \mathcal{L}_{G_v^p}[\beta].\delta$, we update $\mathcal{L}_{G_v^p}[\beta]$ as $\mathcal{L}_{G_v}[\beta]$.

*Step 2.* For each border $\beta'$ in $\mathbb{B}(G_v^p) - \mathbb{B}(G_v)$, if $\exists \beta \in \mathbb{B}(G_v^p) \cap \mathbb{B}(G_v)$, $\mathcal{L}_{G_v^p}[\beta].\delta + \texttt{SPDist}(\delta, \delta') < \mathcal{L}_{G_v^p}[\beta'].\delta$, we update $\mathcal{L}_{G_v^p}[\beta']$ as $\langle \mathcal{L}_{G_v^p}[\beta].\delta + \texttt{SPDist}(\delta, \delta'), \mathcal{L}_{G_v^p}[\beta].\gamma \rangle$.

*Step 3.* If the LNAV of some borders in $G_v^p$ are updated, we require to update the parent of $G_v^p$; otherwise the algorithm terminates.

Iteratively, we perform these steps on the parent node of $G_v^p$ until reaching the root node.

For example, in Figure 4, we show how to add object $m_4$ to vertex $v_7$ from the V-Tree shown in Figure 3. The LNAV tables and distance matrices before adding $m_4$ are shown in Figure 4 (B) and (C), respectively. We first change the status of $v_7$ to *active* and check whether $v_7$ becomes the LNAV of other vertices in $G_4$. $\mathcal{L}_{G_4}[v_6]$ is updated to $\langle 2, v_7 \rangle$. Next, because $v_6$ is a border in $G_4$, $\mathcal{L}_{G_1}[v_6]$ is updated to be $\langle 2, v_7 \rangle$ by lines 9-12 in Algorithm 1. The LNAV of $v_2$ and $v_3$ are refined by lines 13-16 in Algorithm 1. Iteratively, the $\mathcal{L}$ table of $G_0$ (the parent of $G_1$) needs to be updated too. $\mathcal{L}_{G_0}[v_2]$, $\mathcal{L}_{G_0}[v_3]$, and $\mathcal{L}_{G_0}[v_6]$ are directly propagated from $G_1$. $\mathcal{L}_{G_0}[v_9]$ and $\mathcal{L}_{G_0}[v_{13}]$ are refined accordingly. Since $G_0$ is the root, the process terminates.

*2) Removing $m$ from Vertex $v$, $\texttt{Del}(m, v)$:* There are two cases for $v$. $(i)$ After removing $m$ from $v$, $v$ is still an active vertex. The status of $v$ will not change. Thus the LNAV will not change. We only need to remove $m$ from $\mathcal{A}[v]$. $(ii)$ After removing $m$ from $v$, $v$ become an inactive vertex. We remove $m$ from $\mathcal{A}(v)$ and recalculate the LNAV where $v$ was the LNAV for some borders in the subgraphs containing $v$.

Similar to Add (m,v), next we propose a bottom-up method to update LNAV, and the pseudo code is shown in Algorithm 2.

**Updating Leaf Graph $G_v$.** Consider a vertex $u$ in $G_v$ with $\mathcal{L}_{G_v}[u].\gamma = v$. We set $\mathcal{L}_{G_v}[u]$ as $\langle \infty, \phi \rangle$. We find the active vertex $w$ with the minimum distance to $u$, and update $\mathcal{L}_{G_v}[u]$ as $\langle \texttt{SPDist}(w, u), w \rangle$.

**Updating The Ancestor of $G_v$.** Suppose $G_v^p$ is the parent node of $G_v$. Consider a border $\beta$ in $G_v^p$ with $\mathcal{L}_{G_v^p}[\beta].\gamma = v$. We need to update it. We also adopt a 3 step method.

*Step 1.* For each border with $\mathcal{L}_{G_v^p}[\beta].\gamma = v$, we first set $\mathcal{L}_{G_v^p}[\beta] = \langle \infty, \phi \rangle$.

*Step 2.* We recalculate $\mathcal{L}_{G_v^p}[\beta]$ based on borders in $G_v^p$. For each border $\beta'$ in $G_v^p$, say $\beta'$ is the boundary vertex of $G_j, G_j \in \mathcal{C}(G_v^p)$, if $\mathcal{L}_{G_j}[\beta'].\delta + \texttt{SPDist}(\beta', \beta) < \mathcal{L}_{G_v^p}[\beta]$, we update $\mathcal{L}_{G_v^p}[\beta]$ by $\langle \mathcal{L}_{G_j}[\beta'].\delta + \texttt{SPDist}(\beta', \beta), \mathcal{L}_{G_j}[\beta'].\gamma \rangle$.

*Step 3.* If the LNAV of some borders in $G_v^p$ are updated, we require to update the parent of $G_v^p$; otherwise the algorithm terminates.

Iteratively, we perform these steps on the parent node of $G_v$ until reaching the root node.

In Figure 5, we show an example of deleting $m_3$ from $v_8$ on the V-Tree of Figure 2. The leaf subgraph containing $v_8$ is $G_4$. We update LNAV of $v_6$, $v_7$, and $v_8$ with $\langle \infty, \phi \rangle$. And then the LNAV of $v_6$, $v_7$, and $v_8$ are updated to $\langle 10, v_5 \rangle$,$\langle 10, v_5 \rangle$,and $\langle 8, v_5 \rangle$ by the active vertex $v_5$, respectively. We then move on to the parent graph of $G_4$, which is $G_1$. We first reset the LNAV of $v_3$ and $v_6$ to $\langle \infty, \phi \rangle$. Then we find $v_4$ as the LNAV

**Algorithm 2:** $\texttt{Del}(m, v)$

**Input**: $m, v$ // deleting $m$ from vertex $v$

**if** $v$ *only has one active vehicle* **then**
- $v.status \leftarrow$ inactive ;
- **for** *each vertex* $u$ *in* $G_v$ *s.t.* $\mathcal{L}_{G_v}[u].\gamma = v$ **do**
  - $\mathcal{L}_{G_v}[u] \leftarrow \langle \infty, \phi \rangle$ ;
  - **for** *each active vertex* $w \in G_v$ **do**
    - **if** $\mathcal{L}_{G_v}[u].\delta > \texttt{SPDist}(w, u)$ **then**
      - $\mathcal{L}_{G_v}[u] \leftarrow \langle \texttt{SPDist}(w, u), w \rangle$;
- $\texttt{propagation} \leftarrow$ true ;
- **while** $G_v \neq G_0$ *and* $\texttt{propagation}$ **do**
  - $\texttt{propagation} \leftarrow$ false ;
  - **for** *each border* $\beta$ *s.t.* $\mathcal{L}_{G_v^p}[\beta].\gamma = v$ **do**
    - $\mathcal{L}_{G_v^p}[\beta] \leftarrow \langle \infty, \phi \rangle$ ;
    - $\texttt{propagation} \leftarrow$ true ;
    - $d \leftarrow \min_{\beta' \in B(G_j),\ G_j \in \mathcal{C}(G_v^p)} \texttt{SPDist}(\beta', \beta) + \mathcal{L}_{G_j}[\beta'].\delta$;
    - $\mathcal{L}_{G_v^p}[\beta] \leftarrow \langle d, \mathcal{L}_{G_j}[\beta'].\gamma \rangle$;
  - $G_v \leftarrow$ parent of $G_v$ ;

remove $m$ from $\mathcal{A}[v]$;

**Algorithm 3:** $\texttt{knn}(v, k)$

**Input**: $v$: query location; $k$: the number of nearest neighbors
**Output**: $\mathcal{R}$: $k$ nearest objects to $v$

1 Priority queue $\mathcal{R} \leftarrow \Phi$;
2 $\varepsilon$=maximal distance of candidates in $\mathcal{R}$ to $v$(initialized as $\infty$);
3 $u = \texttt{gnav}(v)$;
4 **for** *each active object* $\langle m, \delta \rangle \in \mathcal{A}[u]$ **do**
5   **if** $\texttt{SPDist}(u, v) + \delta < \varepsilon$ **then**
6     $\mathcal{R}.\texttt{Enqueue}(\langle m, \texttt{SPDist}(u, v) + \delta \rangle)$;
7     Update $\varepsilon$;
8 **while** *true* **do**
9   $u = \texttt{nnav}(v, u)$;
10   **if** $\texttt{SPDist}(u, v) \geq \varepsilon$ **then**
11     **break;**
12   **for** *each active object* $\langle m, \delta \rangle \in \mathcal{A}[u]$ **do**
13     **if** $\texttt{SPDist}(u, v) + \delta < \varepsilon$ **then**
14       $\mathcal{R}.\texttt{Enqueue}(\langle m, \texttt{SPDist}(u, v) + \delta \rangle)$;
15       Update $\varepsilon$;
16 return $\mathcal{R}$;

for $v_3$ in $G_1$ through $\mathcal{L}_{G_4}[v_3]\delta = 7$, and $v_4$ for $v_6$ through $\mathcal{L}_{G_3}[v_4].\delta + \texttt{SPDist}(v_4, v_6) = 9$. Iteratively, the LNAV of $G_0$ (the parent of $G_1$) needs to be updated next.

**Correctness of Updating Algorithms.**

*Theorem 1:* The two updating algorithms, Add and Del, correctly maintain the LNAV tables on V-Tree.

**Time Complexity of Updating Algorithms.**

*Lemma 2:* If updating a moving object does not change the state of vertex, the complexity of an update is $\mathcal{O}(1)$. If the updating operation changes the state, the average complexity of an update on V-Tree is

$$\mathcal{O}(\frac{|V| \log \min(|V|, |\mathcal{M}|)}{|\mathcal{M}|}).$$

where $|V|$ is the number of vertices and $|\mathcal{M}|$ is the number of objects.

## IV. kNN Algorithm

### A. Overview of kNN Algorithm

Given a query $q = \langle v, k \rangle$, the kNN query returns top-$k$ active objects ranked by shortest-path distance to $v$. For any active object $m$, we know that $\texttt{SPDist}(m, v) = \texttt{SPDist}(u, v) + \delta(m, u)$, where $u$ is the active vertex of $m$ and $\delta(m, u)$ is the distance from $m$ to $u$. Note that $\delta(m, u)$ is usually much smaller than $\texttt{SPDist}(u, v)$. Based on this observation, we first find the global NAV of $v$, denoted by $u$, and take the active objects of $u$ as the top-$k$ candidates of $v$. Then we mark $u$ as inactive, and find the next global NAV of $v$, $u'$. If $\texttt{SPDist}(u', v)$ is larger than the distance of the $k$-the candidate, denoted by $\varepsilon$ (i.e., $\texttt{SPDist}(u', v) \geq \varepsilon$) we can safely terminate. Next we formally introduce the algorithm.

Suppose we have two functions $\texttt{gnav}(v)$ and $\texttt{nnav}(v, u)$ to find the global NAV and the next global NAV of $v$, respectively. The discovered top-$k$ moving objects are stored in a priority queue $\mathcal{R}$ of size $k$, i.e., it only keeps $k$ objects with the $k$ shortest SPDist to $v$.

Then $\texttt{knn}(v, k)$ works in three steps. $(i)$ It maintains a priority queue with $k$ candidate objects to $v$. Let $\varepsilon$ denote the maximal distances from the candidates to $v$ in the priority queue, which is initialized as $\infty$ (lines 1-2). $(ii)$ It finds the global NAV $u$ of $v$ and $d = \texttt{SPDist}(u, v)$ by calling $\texttt{gnav}(v)$. It adds the active objects into the priority queue (lines 3-7 in Algorithm 3). $(iii)$ It marks $u$ inactive and finds the next global NAV $u$ of $v$ and $d = \texttt{SPDist}(u, v)$ by calling $\texttt{nnav}(v, u)$. If $d > \varepsilon$, which means the active objects of $u$ cannot be kNN result of $v$, and the algorithm terminates (lines 10-11); otherwise, it adds the active objects of $u$ into the priority queue (lines 12-15). Iteratively, the algorithm finds the top-$k$ results.

### B. Function $\texttt{gnav}(v)$

There are two cases for the global NAV of $v$. $(i)$ $v$ and its global NAV are in the same leaf subgraph. We can find the global NAV by exploring all the active vertices in the leaf subgraph and find the nearest one. $(ii)$ $v$ and its global NAV are in two different leaf subgraphs, in which case the shortest path from the global NAV to $v$ must pass-by a boundary vertex of the leaf subgraph or its ancestor subgraphs containing $v$ by Lemma 1. Therefore, we only need to explore the boundary vertices of these subgraphs and find the one connecting the global NAV. In other words, the global NAV of $v$ must be in the local NAV in the nodes from $G_v$ to the root.

Next we discuss how to compute the global NAV from the local NAV. Suppose $G_v$ is the leaf node of $v$. Consider an ancestor of $G_v$, $G_v^a$. For any boundary vertex $\beta$ in $G_v$, let

$$\texttt{LNAVDist}_{G_v^a}(\beta, v) = \texttt{SPDist}(\beta, v) + \mathcal{L}[\beta]_{G_v^a}.\delta$$

denote the local NAV distance from $w = \mathcal{L}_{G_v^a}[\beta].\gamma$ to $v$ in $G_v^a$. The vertex $w$ with the minimal local NAV distance is the global NAV as stated in Lemma 3.

*Lemma 3:* Given a vertex $v$, suppose $G_v$ is its leaf node and let

$$\beta = \arg \min_{u \in \mathbb{B}(G_v^a), u \in B(G_v)} \texttt{LNAVDist}_{G_v^a}(u, v)$$

where $G_v^a$ is an ancestor node of $G_v$. $w = \mathcal{L}_{G_v^a}[\beta].\gamma$ is the global NAV of $v$.

**Function** gnav(v)

**Input**: $v$: query location;
**Output**: the global NAV of $v$

1 $u = \arg\min_{\beta \in \mathbb{B}(G_v)} \texttt{LNAVDist}_{G_v}(\beta, v)$;
2 $\varepsilon \leftarrow \texttt{LNAVDist}_{G_v}(u, v)$;
3 **while** $G_v^p \neq \texttt{NULL}$ *and* $\texttt{SPDist}(G_v^p, v) < \varepsilon$ **do**
4 $\quad u' = \arg\min_{u' \in \mathbb{B}(G_v^p), u' \in B(G_v)} \texttt{LNAVDist}_{G_v^p}(u', v)$;
5 $\quad$ **if** $\texttt{LNAVDist}_{G_v^p}(u', v) < \varepsilon$ **then**
6 $\quad\quad u = u'$;
7 $\quad\quad \varepsilon \leftarrow \texttt{LNAVDist}_{G_v^p}(u, v)$;
8 $\quad G_v^p \leftarrow$ parent of $G_v^p$ ;
9 return $\mathcal{L}_{G_v^p}[u].\gamma$;

Thus a simple method is to enumerate every local NAV in the nodes from $G_v$ to the root. However this method will enumerate every ancestors of $G_v$, and next we propose an efficient method which can skip many unnecessary nodes. To achieve this goal, we have two observations.

**Observation 1. Monotone Non-Decreasing.** Given an ancestor of $G_v$, $G_v^a$, let

$$\texttt{SPDist}(G_v^a, v) = \min_{\beta \in \mathbb{B}(G_v^a)} \texttt{SPDist}(\beta, v).$$

We have $\texttt{SPDist}(G_v, v) \leq \texttt{SPDist}(G_v^a, v)$ as stated in Lemma 4.

*Lemma 4:* Given two nodes $G_v$ and $G_v^a$, where $G_v$ is the leaf node of $v$ and $G_v^a$ is an ancestor of $v$, we have

$$\texttt{SPDist}(G_v, v) \leq \texttt{SPDist}(G_v^a, v).$$

**Observation 2. Early Termination.** Let

$$\texttt{LNAVDist}(G_v^a, v) = \min_{\beta \in \mathbb{B}(G_v^a)} \texttt{LNAVDist}_{G_v^a}(\beta, v).$$

Consider two ancestors of $G_v$, $G_v^p$ and $G_v^a$, where $G_v^a$ is the parent of $G_v^p$.If the LNAV distance of $G_v^p$ ($\texttt{LNAVDist}(G_v^p, v)$) is not larger than $\texttt{SPDist}(G_v^a, v)$, i.e.,

$$\texttt{LNAVDist}(G_v^p, v) \leq \texttt{SPDist}(G_v^a, v)$$

we can skip $G_v^a$ and its ancestors, because the vertices in them have larger distance based on the monotone non-decreasing property and Lemma 3.

Based on these two observations, we propose a bottom-up method to explore the borders of subgraphs containing $v$, which is shown in Function gnav.

**Exploring The Vertices in Leaf Graph $G_v$.** Suppose $G_v$ is the leaf subgraph containing $v$. Consider an active vertex $u$ in $G_v$, $u$ is also the NAV of itself in $G_v$, so we compute the vertex $u$ in the leaf with the minimal distance (line 2 in Function gnav).

**Exploring The Borders in Ancestors of $G_v$.** Let $G_v^p$ be the parent subgraph of $G_v$. $\varepsilon$ holds the minimum of $\texttt{LNAVDist}_{G_v}(u, v)$. If $\texttt{SPDist}(G_v^p, v) \geq \varepsilon$; the algorithm terminates. On the contrary, if $\texttt{SPDist}(G_v^p, v) < \varepsilon$, we find GNAV in $G_v^p$and compute the vertex with the minimal LNAV in $G_v^p$. If there is a vertex in $G_v^p$ with smaller distance, we select the vertex with the minimal distance (lines 4-7).

For example, in Figure 6 we illustrate how to search the global NAV of $v_7$ on the V-Tree shown in Figure 3. $G_4 =$ Leaf$(v_7)$ and $v_8$ is the NAV to $v_7$ in $G_4$ with $\texttt{SPDist}(v_8, v_7)$=2. The parent graph of $G_4$ is $G_1$. Since $\texttt{SPDist}(G_1, v_7) = 2$, any active vertex that passes borders in $G_1$ and its ancestor graphs to $v_7$ must have a distance greater or equal to 2. The algorithm terminates and returns $v_8$ as the global NAV for $v_7$.

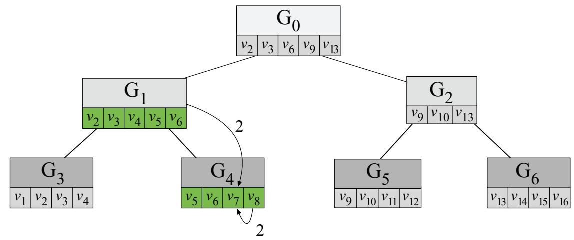



Fig. 6. Searching the global NAV for $v_7$

**Function** nnav$(v, u)$

**Input**: $v$: query location; $u$ the previous returned global NAV of $v$
**Output**: the next global NAV of $v$

1 $\widetilde{G} \leftarrow \texttt{Inactivate}(u)$ ;
2 return gnav$(v)$ ;

**Calculating $\texttt{SPDist}(\beta, v)$ efficiently.** Given two ancestor graphs $G_m$ and $G_n$, $G_n = PG(G_m)$, the SPDist from the boundary vertices of $G_n$ to $v$ are computed based on the SPDist from the boundary vertices of $G_m$ to $v$, i.e., if $\beta \in B(G_n)$,

$$\texttt{SPDist}(\beta, v) = \min_{\beta_j \in B(G_m)} \texttt{SPDist}(\beta, \beta_j) + \texttt{SPDist}(\beta_j, v).$$

Hence, the SPDist from the borders of $G_m$ to $v$ are repeatedly used when computing the SPDist from the borders of $G_n$ to $v$. We can utilize the dynamic programming to avoid duplicated computation.

## C. *Function* nnav$(v, u)$

Suppose $u$ is the global NAV of $v$ found by the last gnav or nnav call. The function nnav$(v, u)$ finds the next GNAV of $v$ by two steps: $(i)$ inactivating $u$ on V-Tree, and $(ii)$ finding the next GNAV by calling gnav$(v)$, as shown in Function nnav.

**Inactivating $u$ on V-Tree.** In order to find the next GNAV of $v$ based on the LNAV tables maintained on V-Tree, we need to change the status of $u$ from *active* to *inactive* and let the borders who has $u$ as their local LNAV finds new ones. Note that, this procedure is identical to function Del as if we delete the last moving object of $u$. However, we cannot directly update the LNAV tables on V-Tree when answering a kNN query due to concurrency control issues. Instead, for each query we maintain a local buffer $\widetilde{G}$ to save the updated LNAV entries. Let Inactivate$(u)$ be the function to update LNAV tables as if $u$ changes from *active* to *inactive*, and the updated entries are saved in $\widetilde{G}$ (line 1 in Function nnav). $\mathcal{L}_{\widetilde{G}}$ will first return values from the local buffer if the local buffer has them, and from the V-Tree otherwise.

**Finding The Next GNAV.** We directly call gnav$(v)$ on $\widetilde{G}$ with the updated LNAV tables after inactivating $u$ (line 2). gnav$(v)$ returns the current GNAV of $v$.

nnav$(v)$ works on a slightly modified $\widetilde{G}$ and explores from the leaf node $G_v$ containing $v$ to its ancestor graphs until the global NAV of $v$ is found. Hence, we may explore unchanged

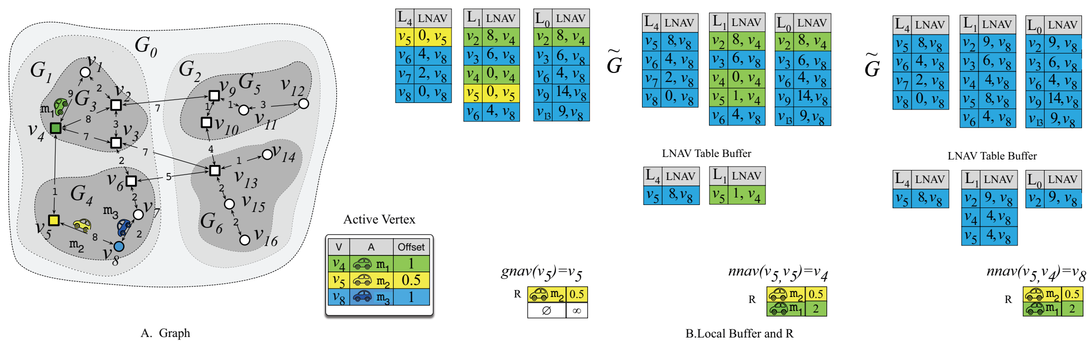


Fig. 7. Progressing of $\texttt{knn}(v_5, 2)$

subgraphs multiple times. To avoid such repeated calculation, we introduce a priority queue $Q$ to record the explored borders and their `LNAVDist` in functions `gnav` and `nnav`. We keep the top entry of $Q$ updated with the most recent $\widetilde{G}$ and the updates of other non-top entries can be delayed (other non-top entries in $Q$ are irrelevant to the final result this time, because `LNAVDist` of a border can only be increased on the most recent $\widetilde{G}$). We also record the subgraph $G_s$ where we stopped in the last `gnav` or `nnav` call. If the early termination condition does not hold anymore, i.e., the current shortest distance in $Q$ becomes larger than $\texttt{SPDist}(G_s, v)$, we need to resume the exploration from $G_s$ and continue with its ancestor graphs until the global `NAV` of $v$ is found.

For example, we show $\texttt{knn}(v_5, 2)$ in Figure 7. We show the graph and the active object table in Figure 7 (A), and the `LNAV` tables and $R$ after each iteration in Figure 7 (B), where the local `LNAV` buffer only keeps the updated entries. In the first iteration, $\texttt{gnav}(v_5) = v_5$. We add $\langle m_2, 0.5\rangle$ to $R$ and $\varepsilon = \infty$. In the second iteration, $\texttt{nnav}(v_5, v_5) = v_4$. We add $\langle m_1, 2\rangle$ to $R$ and $\varepsilon = 2$. In the third iteration, $\texttt{nnav}(v_5, v_4) = v_8$. Since $\texttt{SPDist}(v_8, v_5) > \varepsilon$, the algorithm terminates.

### D. *Correctness of* `knn`

*Theorem 2:* Functions `gnav`, `nnav`, and algorithm `knn` correctly find the global `NAV`, the next global `NAV`, and the $k$ nearest moving objects of $v$, respectively.

### E. *Time Complexity of* `knn`

*Lemma 5:* Given a graph $G$ and `V-Tree`, which contains $|V|$ vertices and $|\mathcal{M}|$ moving objects on the road network. Assume moving objects are uniformly distributed on the road network, the average time complexity of kNN search is $\mathcal{O}(\frac{k \cdot |V| \cdot \log min(|V|, |\mathcal{M}|)}{|\mathcal{M}|})$.

## V. Experiment

We evaluated the performance of `V-Tree` and compared with baseline approaches, including index construction, $k$NN queries, updates, and scalability of `V-Tree`.

**Datasets: Road Network.** We used seven real-world road networks, which were widely used in previous studies [31], [16], [25]. The size of the datasets varied from 21,048 to 24 million vertices, as illustrated in Table II.

**Datasets: Query and Object.** We used two real-world datasets `BJTaxi` and `SpecialCar`. `BJTaxi` was obtained from real taxi trajectories in Beijing, which contained 28,000 taxi trajectories from 8:00am to 9:00am on Sept. 30, 2015. `SpecialCar` contained trajectories of 16,000 cars from 9:30pm to 10:30pm on July 15, 2015, which was gotten from a company like Uber. We first mapped the positions of moving objects and queries onto the road network using existing methods [4], [24], [19]. The moving objects updated locations in every second. We showed the number of moving objects, queries per hour, and queries per second of `BJTaxi` and `SpecialCar` in Table III.

For other six datasets, we synthetically generated moving objects and queries. We randomly selected one percent of vertices as the initial positions of moving objects, and moving objects were moving following the same distribution draw from `BJTaxi` and `SpecialCar`. For queries, we generated the queries similar to `BJTaxi` and `SpecialCar` (i.e., the number of generated queries per second and the number of active objects). The positions of these queries were randomly selected from the vertices of each dataset, and they evenly arrived during each hour. The detailed statistics were illustrated in Table III.

**Baseline.** We compared our `V-Tree` with three state-of-the-art methods, SILC[25], ROAD[16], and G-Tree[31]. The implementations of ROAD and G-Tree were provided by the authors, SILC was implemented by ourselves. As the SILC, ROAD, G-Tree do not support kNN query on moving objects directly, we extended them to support our problem as follows. We first built indexing structures for a (static) road network. Then we created an occurrence index, which keeps a map from a vertex to a list of moving objects that belong to this vertex. If a vertex has some moving objects, we call it an active vertex. Next, given a query we computed kNN active vertices using existing techniques, and added the moving objects belonging to such vertices into the results. Note that we may not need to find $k$ active vertices because an active vertex may contain multiple moving objects and the algorithm will stop when finding $k$ moving objects. For example, G-Tree used a best-first algorithm to find kNN active vertices by traversing the tree from the root. We kept an occurrence index for each tree node. If a tree node had no moving object, we pruned the node; otherwise we accessed its children to compute the active vertices. SILC also used a best-first method to compute kNN results. We also utilized the occurrence index to keep the active vertices, computed top-$k$ active vertices and added moving objects of active vertices into the result set. In ROAD, we also only considered active vertices based on the occurrence index and prune other vertices. When the moving objects were updated, we only updated the occurrence index and recomputed the top-$k$ results using the static index and the updated occurrence index. For G-Tree, we set the leaf capacity $\tau$=32 and the fanout $f$=4. For ROAD, we set the hierarchy level $l = 8$ and fanout $f$=4.

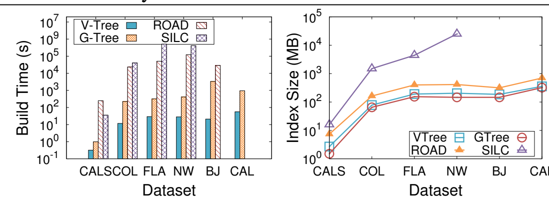
**Metrics.** Suppose there were $n$ queries $q_1 \cdots q_n$ in a period. Before executing $q_i$, we needed to update objects. We used the

TABLE II. CHARACTERISTICS OF ROAD NETWORKS

| Database | Description | Vertices | Edges |
|---|---|---|---|
| BJ | Beijing | 1,278,984 | 2,677,984 |
| CALS | California | 21,048 | 43,386 |
| COL | Colorado | 435,666 | 1,057,066 |
| FLA | Florida | 1,070,376 | 2,712,798 |
| NW | Northwest USA | 1,207,945 | 2,840,208 |
| CAL | California and Nevada | 1,890,815 | 4,657,742 |
| USA | Full USA | 23,947,347 | 58,333,344 |

TABLE III. CHARACTERISTICS OF QUERIES AND OBJECTS

| Database | Source | Objects | Queries per Second | Updates per Second |
|---|---|---|---|---|
| BJTaxi | real-world | 28000 | 6 | 75 |
| SpecialCar | real-world | 16000 | 4 | 60 |
| CALS | synthetic | 210 | 1 | 1 |
| COL | synthetic | 4357 | 1 | 27 |
| FLA | synthetic | 10704 | 2 | 66 |
| NW | synthetic | 12079 | 3 | 174 |
| CAL | synthetic | 18908 | 2 | 116 |
| USA | synthetic | 239473 | 94 | 1469 |




(a) Construction Time (b) Index Size

Fig. 8. Index Comparison

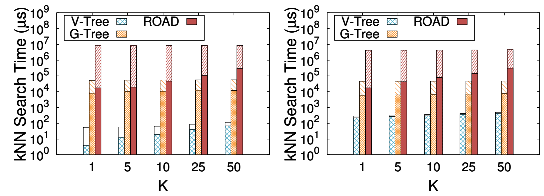
amortized time $T_a = \frac{T_u + \sum_{i=1}^{n} T_{q_i}}{n}$, where $T_u$ and $T_q$ were the update and query time, respectively.

**Environment.** We conducted experiments on a 64-bit Linux computer with Intel 3.10GHz i5-3450 CPU and 32GB RAM. Our program was compiled with G++ 4.9 using the O3 flag. We did not parallelize the program and used one core.

### A. Evaluation on Index Construction

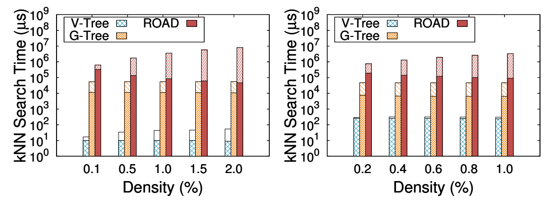
We conducted experiments by varying two parameters: fanout $f$ and the maximum number of vertices in a leaf node $\tau$, and the details are in our technical report [1].

We evaluated the V-Tree index building time and space overhead with G-Tree, SILC, and ROAD on the datasets CALS, COL, FLA, NW, BJ, and CAL. V-Tree consists of tree structure, distance matrices, LNAV tables, and active object tables. Note that the index time of SILC and ROAD was too long for BJ and CAL, and the results were not reported. As can be seen from Figure 8, the time of V-Tree was better than that of SILC and ROAD by almost 3 orders of magnitude, and 1 order of magnitude faster than G-Tree in all the 6 test datasets. For example, on FLA, the construction time of V-Tree was only 29 s, G-Tree consumed 320 s, SILC required 142 hours, and ROAD cost 14 hours. The main reason was that (1) SILC required to compute the shortest path of every two vertices which was rather time consuming, (2) ROAD had to compute and store shortest-path distances of all border pairs, (3) our bottom-up construction method can reduce many unnecessary computations than G-tree. For index size, V-Tree outperformed SILC and ROAD by 1 order of magnitude, and achieved almost the same result as G-Tree. This is because (1) the space overhead of SILC is $\mathcal{O}(|V|^{1.5})$ and it was rather expensive to compute all-pair shortest paths, and (2) ROAD computed larger numbers of borders than our methods and needed to store shortest-path distances of all border pairs. Thus SILC and ROAD took more space and time than ours.




(a) BJTaxi Dataset (b) SpecialCar Dataset

Fig. 9. Varying K on BJTaxi and SpecialCar Datasets




(a) BJTaxi Dataset (b) SpecialCar Dataset

Fig. 10. Varying Density on BJTaxi and SpecialCar Datasets

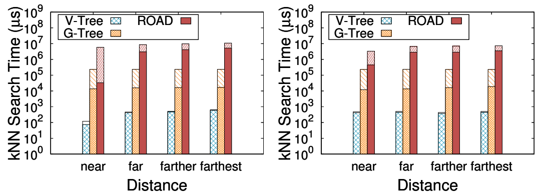
### B. kNN Query

We compared our proposed method with state-of-the-art approaches. The performance of kNN was evaluated in two ways. The first is the performance on real-world datasets, where we used the datasets BJTaxi and SpecialCar containing real trajectories of moving objects and request queries on road network of BeiJing. As SILC is too expensive and not able to generate the road index for BJ, we omit this algorithm on real-world datasets. The other is to evaluate the performance on varying datasets of synthetic data. As mentioned, we used the ROAD method with the best performance parameter $t = 4$, and $l = 8$ to generate road index. For V-Tree and G-Tree, we set $f = 4$, and $\tau = 32$.

We evaluated the kNN search efficiency of V-Tree, and used SILC, ROAD, G-Tree as baselines. We varied $k$, object distribution, datasets, object distance, and updating interval on real-world road networks.

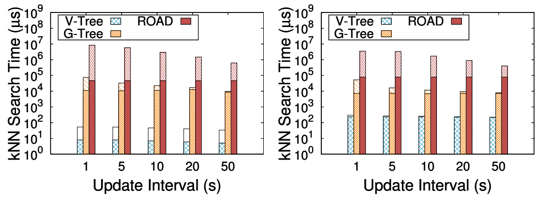
**Varying k:** We evaluated the performance of V-Tree, ROAD, and G-Tree for kNN query on real-world dataset of taxi and SpecialCar. We varied $k$ in 1, 5, 10, 20, 50. We reported the average time on the real datasets, including kNN search and update time. As can be seen from Figure 9, the whole histogram is the amortized time, and the bottom part is the average single query time and the top part is the average update time. We had the following conclusions. Firstly, the average time of kNN query of V-Tree is the fastest one among the three algorithms. The amortized query time of V-Tree was 65 $\mu s$ for $k = 10$, while ROAD took nearly $10^7$ $\mu s$ and G-Tree took $10^5$ $\mu s$. V-Tree method outperformed ROAD by almost 5 orders of magnitude and outperformed G-tree by 2-3 orders of magnitude. Secondly, the single query time of V-Tree was also the best one in the three algorithms. When $k$ was 10, the average query time of V-Tree was 19 $\mu s$ while ROAD took $10^5$ $\mu s$ and G-Tree took $10^4$ $\mu s$. This was attributed to the lower updating cost and the efficient query method. ROAD took much time because it involved many times of Dijkstra algorithms to compute the shortest-path distance. G-tree was expensive for updates on moving objects.

**Varying Density:** We evaluated the performance on varying object densities in Figure 10. There are 28000 taxis and 16000 Special cars, and the number of vertices was 1,278,984. We select 0.1%, 0.5%, 1.0%, 1.5% and 2.0% portion of objects for BJTaxi; 0.2%, 0.4%, 0.6%, 0.8% and 1.0% portion of



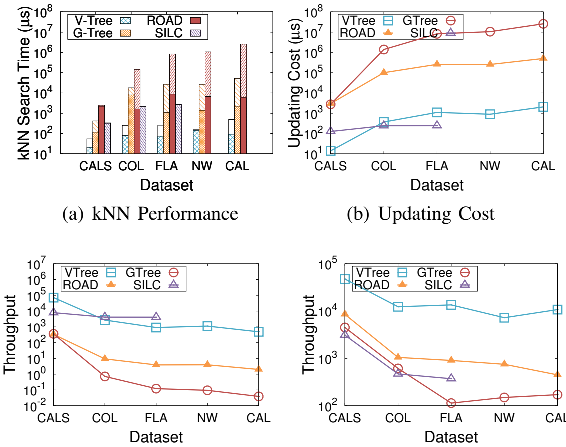

(a) BJTaxi Dataset (b) SpecialCar Dataset

Fig. 11. Varying Distance on BJTaxi and SpecialCar Datasets




(a) BJTaxi Dataset (b) SpecialCar Dataset

Fig. 12. Varying Updating Interval on BJTaxi and SpecialCar Datasets




(a) kNN Performance (b) Updating Cost

(c) Updating Throughput (d) Query Throughput

Fig. 13. Performance on Varying Datasets

objects for `SpecialCar`. The experiment was conducted by 3,600 queries, and we set k = 10 and evaluated the average time. We made the following three observations. Firstly, `V-Tree` outperformed state-of-the-art G-Tree and ROAD by 2-3 orders of magnitude for the search time. Secondly, with the increasing of searched objects, the amortized search time of the three algorithms all increases. This is because the overhead of updating increased as the number of objects grew. Thirdly, the single query time of three algorithms all decreased. Because of small number of objects and the sparse distribution of the vertices in the graph, the searching space for the k nearest objects was increasing by the density decreasing. The cost of computation will increase by the farther vertices. The updating cost of `V-Tree` increased very slightly, which was more suitable for moving objects indexing.

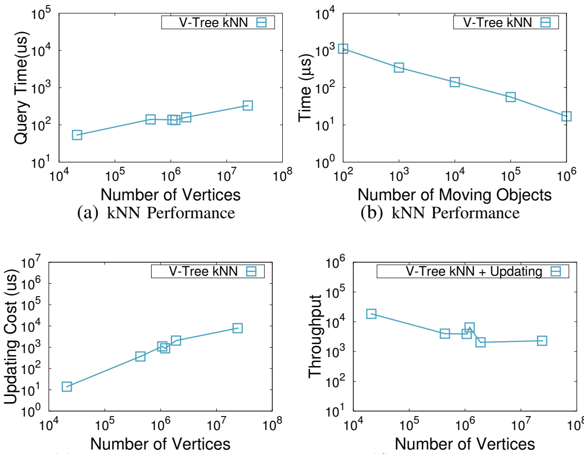
**Varying Distance:** We evaluated the performance on varying object distances in Figure 11. To generate the objects with varying distances, we first extracted the query position from the real queries. Then we sorted the objects by their distances to the query position. The objects were partitioned into 4 equally sized datasets, by their distance, denoted as near, far, farther, and farthest. We can see that the `V-Tree` was almost stable when the distance changed, and outperformed ROAD and SILC and G-Tree by 2-6 orders of magnitude. ROAD and SILC needed to compute more vertices for the long distance objects; while on the contrary `V-Tree` structure can acquire the kNN vertices by accessing fewer tree nodes, no matter how far away the objects were. This means that the method of `V-Tree` kNN method is suitable for the different distance of query.

**Varying Update Interval:** We evaluated the amortized query time of the algorithms on two real-world datasets `BJTaxi` and `SpecialCar`. We set $k$ = 10 and. We used the positions of moving objects and queries in this period to evaluate the updating cost. As illustrated in Figure 12, we selected the update interval 1 s, 5 s, 10 s, 20 s, and 50 s. We know that the higher frequency of updating, the fewer updating objects in the updating period. We can see that with the update frequency increasing, the cost of amortized query time declined. The reason is the updating cost declines as the longer updating period amortized updating interval. Meanwhile, when updating interval is 1 $s$, the amortized update time of `V-Tree` on `BJTaxi` and `SpecialCar` is 44$\mu s$ and 263 $\mu s$ respectively.

**Various Datasets:** We evaluated the performance of the algorithm on five datasets CALS, COL, FLA, NW and CAL as illustrated in Table III and set $k = 10$. The number of vertices varied from $21,048$ to $1,890,815$. As can be seen from Figure 13, The amortized time of `V-Tree` significantly outperformed G-Tree, ROAD, by more than 3 orders of magnitude, and more than one order of magnitude compared with SILC. The single query time of `V-Tree` outperformed other algorithms by at least one order of magnitude. `V-Tree` not only had small kNN search time, but also had small preprocessing time. Note that although the SILC algorithm was the fastest one of the other algorithms, the index size was too huge, e.g. 24GB for NW. In contrast, the size of `V-Tree` was only 220MB. Since SILC consumed too large amount of memory and pre-processing time, we did not evaluate it on BJ and CAL datasets. The superiority of throughput of `V-Tree` is shown in Figure 13(c) and Figure 13(d). In addition, the throughput (including search and update) was slightly decreased with the increase of dataset.

## C. Scalability

We evaluated the scalability of `V-Tree` from three aspects, indexing, search and update. Figure 14 showed the result. Firstly, we evaluated the time and space scalability of `V-Tree`. We set $k = 10$ and $|\mathcal{M}| = 0.01|V|$, and the moving objects was uniformly distributed. We calculated the average overhead of 10K random kNN queries. We can see from 14(a) that `V-Tree` had very good scalability as the data size increased from $10^4$ to $10^8$. The average search time on the US road network with 24 million vertices was only 330.96 $\mu s$. Secondly, we evaluated the scalability of the number of moving objects on road network. We randomly generated uniformly distributed vertices as the number of moving objects. We set $k$ = 10, and used BJ. As can be seen form Figure 14(b), searching time of kNN query decreased by the number of the objects. This is for the more objects were indexed, the vertex will be more likely to not change its state between *active* and *inactive*, which means that the influence of moving objects will decrease. This means that our method was suitable for huge number of moving objects indexing on road networks. Thirdly, the throughput (including update and search) slightly decreased with increase of vertices. Fourthly, Table IV showed that the index size of `V-Tree` was scalable. We evaluated the index size for different datasets, and the number of vertices increased from 21 thousands to 24 millions. We can see that the index size of `V-Tree` increased almost linearly as the size




(a) kNN Performance (b) kNN Performance (c) Updating Performance (d) Query Throughput

Fig. 14. Scalability of V-Tree

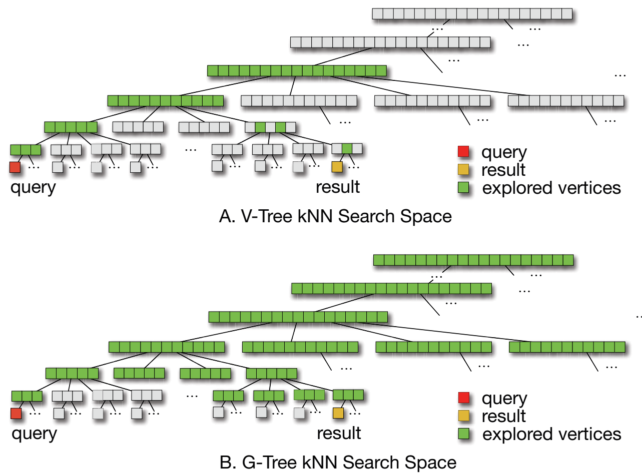



Fig. 15. V-Tree and G-Tree kNN Search Space Overview

of vertices and edges increased.

TABLE IV. INDEX TREE SIZE OF V-TREE

| Size | CALS | COL | NW | FLA | BJ | CAL | USA |
|---|---|---|---|---|---|---|---|
| Vertices | 21K | 435K | 1.2M | 1.1M | 1.2M | 1.8M | 24M |
| Edges | 43K | 1.1M | 2.8M | 2.7M | 2.7M | 4.7M | 58M |
| Data | 702K | 19M | 46M | 48M | 81M | 87M | 1270M |
| V-Tree | 3.5M | 91M | 220M | 224M | 233M | 416M | 5639M |

## VI. RELATED WORKS

### A. Single Pair Shortest Path Queries

The single pair shortest path (SPSP) queries, which find the shortest path for two vertices on the road networks, have been extensively studied, e.g.,G-Tree[31][30], HEPV[12][13], HiTi[14], TNR[3], CH[7] and PHL[2]. HEPV[12][13] partitions the graph by cutting vertices and pre-computes all the shortest paths between all border pairs. It is both time consuming and space consuming to store all such pairs, and cannot support large graphs. Furthermore, HEPV only considers three layers and it is not clear how to extend it to support multiple layers. HiTi[14] computes the shortest paths for objects in different subgraphs by using the A* algorithm. It utilizes the Euclidean distance to estimate a lower bound of the road-network distance and then uses the A* algorithm to prune subgraphs withlarger distances. Transit Node Routing (TNR)[3] calculates the distances of each vertex to a set of transit nodes and utilizes the transit nodes to compute the shortest-path distance. Contraction Hierarchies (CH)[7] first pre-computes the road network by appending additional edges. Then bidirectional shortest-path search is used to restrict the edges leading to more important nodes which reduces the search spaces. The Pruned Highway Labeling (PHL)[2] uses highway-based labeling and a preprocessing algorithm to compute the distance. G-Tree[31], [30] is a hierarchy structure to do kNN query, which also supports SPSP query. An assembly-based method is proposed to efficiently compute the shortest path between two vertices. These algorithms cannot support dynamical updates of moving objects either.

### B. kNN Query on Road Networks

Recently, kNN query on road networks has been extensively studied, such as INE[22], IER[22], ROAD[16], SILC[25], G-Tree[31][30]. Incremental Euclidean Restriction(IER)[22] uses Euclidean distance as a pruning bound to acquire the kNN results. Incremental Network Expansion(INE)[22] improved IER Euclidean distances bound by expending searching space from the query location. SILC[25] precomputed the shortest paths between all possible vertices in the network and then made use of Quadtree-based encoding method to reduce the storage cost. Route Overlay and Association Directory(ROAD)[16] and G-Tree [31], [30] extend HiTi and HEPV to support kNN search. ROAD uses a hierarchical structure, which recursively partitions the whole graph to a hierarchy of interconnected regional sub-networks. The *shortcuts* between the partitioned vertices are precomputed. It uses the INE-like method to compute the kNN results by using shortcuts to compute a tighter bound. G-Tree is a hierarchy structure to compute kNN results on road networks. It also uses a hierarchical structure and adopts an assembly-based method to efficiently compute kNN results. Different from our method, these approaches assume that the locations of objects are not frequently changed. In our problem, obviously the locations of objects are dynamically changed and thus existing algorithms cannot efficiently support our problem.

V-Tree extends G-Tree to support moving objects. G-Tree is better than ROAD because G-Tree uses the dynamic-programing algorithm to compute the shortest-path distance for two objects across different subgraphs while ROAD uses the Dijkstra algorithm to compute the shortest paths based on the boundary nodes in subgraphs. SILC takes $|V|^{1.5}$ space and could not support large graphs. Next we explain why V-Tree is much better than G-Tree. Firstly, in G-Tree, it can only know which subtrees contain active vertices, and it needs to use the dynamic-programming algorithm to compute the distance for objects in different subgraphs. In V-Tree, we utilize the LNAV structure to keep the nearest active vertex for each border. If the query and the active vertex are in the same node, it can directly retrieve the active vertex. If the query and the active vertex are not in the same node, it can utilize the LNAV structure to efficiently find the active vertex. In this way, we can avoid duplicated computation using the LNAV structure. Secondly, G-Tree uses a top-down manner to traverse the tree structure and it may visit unnecessary nodes as shown in Figures 15. V-Tree uses a down-up manner, and it only visits the active vertices based on the LNAV structure. Thus the search space of V-Tree is much smaller than G-Tree.

### C. Moving Objects Query

There are some studies on finding kNN moving objects with Euclidean distance, e.g., TPR-Tree[26], $B^x$-Tree[11], STR-Tree[23], TB-tree[23], DSI[29], $V^*$kNN[21], MOVNet[27]. The Time-parameterized R-tree (TPR-tree) [26] extends R-tree to index moving point objects by Euclidean distance. The Spatio-Temporal R-tree (STR-tree) [23] extends R-tree to index spatio-temporal objects. Trajectory-Bundle tree (TB-tree) uses a hybrid structure to process the trajectories of moving objects. $B^x$-Tree[11] enables the $B^+$-Tree to support range query and $k$NN queries and continuous queries. Shortest-Distance-based Tree(SD-Tree)[28] reduces the continuous query update cost by precomputing some vertices distances. Dynamic Strip Index(DSI)[29] uses the strip index structure to support distributed processing on kNN queries of moving objects. MOVNet[27] uses an in-memory grid structure to index moving objects. These studies focus on identifying kNN results for continuous queries. However they cannot support kNN search on road-network distances.

### D. Continuous Query

Some studies focus on continuous k-NN queries [20], [17], DLMTree[8], COMET[6], which study how to continuously answer a query, but they do not focus on efficient kNN search on moving objects in large-scale road networks. Specifically, [20] uses an incremental monitoring algorithm and a group monitoring algorithm to share the computation on moving objects. [17] utilizes the driving directions and speeds to reduce unnecessary computations. DLMTree[8] focuses on continuous reverse k nearest queries in road networks. COMET[6] proposes a collaborative framework that combines different techniques, e.g, safe segment and influence segment, to reduce the search space. Thus they focus on optimizing a continuous query and do not emphasize on optimizing kNN search for a large number of online queries in road networks.

## VII. Conclusion

In this paper we have studied the kNN search on moving objects with road-network constraints. We proposed a novel tree structure `V-Tree`. We devised an efficient algorithm to construct the index. We developed efficient strategies to support updates of moving objects. We designed an efficient kNN search algorithm using `V-Tree`. Experimental results show that our method significantly outperforms baseline algorithms.

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